	MIDTERM EXAM 1	
Name, Surname:	Department:	GRADE
Student No:	Course: Calculus I	
Signature:	Exam Date: 14/11/2019	

Duration is 70 minutes. You can not use L'hopital's rule.

1. (A) (15 pt)$\lim_{h\to 0} \frac{(x+h)^{10}-x^{10}}{h} = \boxed{}$

Solution: This is the definition of derivative of $y = x^{10}$. The answer is $10x^9$.

(B) (15 pt) $\lim_{x\to 0} \frac{6x}{5\sin x + 2x} = \boxed{}$

Solution:

$$\lim_{x\to 0} \frac{6x}{5\sin x + 2x} = 6\frac{1}{5\lim_{x\to 0} \frac{\sin x}{x} + 2} = \frac{6}{5+2}$$

(C) (15 pt) $\lim_{x\to -\infty} \sqrt{x^2+3x} + x = \boxed{}$

Solution:

$$\lim_{x\to -\infty} \sqrt{x^2+3x} + x = \lim_{x\to -\infty} \frac{x^2+3x-x^2}{\sqrt{x^2+3x}-x} = \lim_{x\to -\infty} \frac{3x}{|x|\sqrt{1+\frac{3}{x}}-x} = \lim_{x\to -\infty} \frac{3x}{-x\sqrt{1+\frac{3}{x}}-x}$$

$$= \frac{3}{-\lim_{x\to -\infty}\sqrt{1+\frac{3}{x}}-1} = \frac{3}{-2}.$$

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2. (15 pt) Using **implicit derivative**, find an equation of tangent line of the curve defined by $\frac{\pi}{2}\tan(xy) - \frac{\pi}{2} = 0$ at $(x, y) = (\pi, \frac{1}{4})$. $\boxed{}$

Solution: Take the derivative of both sides:

$$\frac{\pi}{2}(1+\tan^2(xy))(y+xy') = 0 \implies y + xy' = 0 \implies y' = \frac{-y}{x}$$

At the given point:

$$y' = \frac{-1}{4\pi}$$

Tangent line is

$$y - \frac{1}{4} = \frac{-1}{4\pi}(x-\pi)$$

3. (15 pt) Show that the equation $1+\sin x = x$ has at least one solution.

Solution: Let $f(x) = 1+\sin x - x$. $f(0) = 1$, $f(2\pi) = 1+0-2\pi$. Since $f(0) > 0$, $f(2\pi) < 0$ and f is continuous everywhere, there is $c \in (0, 2\pi)$ such that $f(c) = 0$. That is $1+\sin c - c = 0$.

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4. (15 pt) Find the derivative of $f(x) = (xe^{\cos x}+1)^3$ at $x = \frac{\pi}{2}$.

Solution:

$$f'(x) = 3(xe^{\cos x}+1)^2(e^{\cos x}+xe^{\cos x}(-\sin x))$$

Use $\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$, $e^0 = 1$ to get

$$f'(\pi/2) = 3\left(\frac{\pi}{2}+1\right)^2\left(1-\frac{\pi}{2}\right)$$

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5. (15 pt) Find the derivative of $(f\circ g)'$ at $x = -1$ where $f(u) = e^{u^2+2u}$, $u = g(x) = \frac{1}{x^2} - 1$.

Solution:

$$f'(u) = e^{u^2+2u}(2u+2), \qquad g'(x) = \frac{-2}{x^3},$$

$$g(1) = 0, \quad g'(-1) = 2, \quad f'(g(-1)) = f'(0) = 2$$
$$(f\circ g)'(-1) = f'(g(-1))g'(-1) = 4$$